QUANTUM PHYSICS AND THE IDENTITY OF INDISCERNIBLES*

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Abstract This paper is concerned with the question of whether atomic particles of the same species, i.e. with the same intrinsic state-independent properties of mass, spin, electric charge, etc, violate the Leibnizian Principle of the Identity of Indiscernibles, in the sense that, while there is more than one of them, their state-dependent properties may also all be the same. The answer depends on what exactly the state-dependent properties of atomic particles are taken to be. On the plausible interpretation that these should comprise all monadic and relational properties that can be expressed in terms of physical magnitudes associated with self-adjoint operators that can be defined for the individual particles, then the weakest form of the Principle is shown to be violated for bosons, fermions and higher-order paraparticles, treated in first quantization.

1. Introduction

In the philosophical literature the notions of identity, individuality and indistinguishability have received a great deal of attention. In this paper we shall discuss how these notions have been treated by quantum physicists, and discuss the relevance of this to the traditional philosophical arguments about these matters.

Identity is a relation that may exist between items that figure in a particular area of discourse. Item *a* is *identical* with item *b*, symbolically *a*=*b*, means informally that there are not in reality *two* distinct items at all, but only *one* item, which may be referred to indifferently as *a* or *b*. We use 'item' here as a neutral word that comprehends both *universals* and *particulars*. Particulars that *exist* in the physical world we shall refer to as *physical individuals* or just individuals for short. The existence of such individuals we take to imply their persistence through time. They are *continuants*. Physical individuals are to be distinguished then from other sorts of particular such as events or states of affairs that may be said to *occur* or *obtain* rather than to exist. In the philosophical literature individuals which have more or less well-defined spatial locations are often referred to as 'things'.

What confers particularity, or individuality as we shall call it, on physical individuals? This raises the fundamental problem of how the particular is related to the universal, of how an individual is related to its attributes. Is it possible for two (non-identical) individuals to have all the same attributes in common, that is to say to be indistinguishable (indiscernible in traditional philosophical terminology)? Leibniz famously claimed that this was not possible, his Principle of the Identity of Indiscernibles (PII). In terms of second-order logic with equality PII states

$$\forall F\ (F(\underline{a}) \longleftrightarrow F(\underline{b})) \rightarrow \underline{a} = \underline{b} \qquad *$$

where a and b are any two individual constants and F is a predicate variable ranging over the possible attributes of these individuals.

$*$ should be contrasted with the Principle of the Indiscernability of Identicals.

$$\underline{a} = \underline{b} \rightarrow \forall F(F(\underline{a}) \longleftrightarrow F(b)) \qquad **$$

$**$ is uncontroversial, provided the attributes do not involve intensional contexts. If two individuals are identical so there is in reality just one individual, then there can only be one set of truly predicated attributes. But $*$ has led to a great deal of argument. What sort of attributes should be included in the range of the variable F? Should we include relations (non-monadic properties)?[1] If we include the attribute 'being indentical with a', which is certainly true of a, then $*$, is a theorem of second-order logic. But suppose we rule out trivializations of PII of this sort, on the grounds that such an attribute is not a genuine monadic property, but expresses a relation of a to itself which relation (of identity) is also satisfied by b, then we can still distinguish a weak and strong version of PII.

Weak version : F includes properties of spatial location.

Strong version : F excludes properties of spatial location.

Leibniz himself apparently subscribed to the strong version of PII. Hence his interest in searching for indistinguishable leaves in the Herrenhausen Gardens in Hanover[2]. If we subscribe to PII in the weak version this raises important questions

concerning what we are to understand by spatial location. On a relational theory of space a circularity threatens. Individuation of material objects-leaves, tables, chairs-involves specifying their location in space. But this location involves their relations with other physical individuals comprising a reference frame. Unless the world is sufficently asymmetrical that each object always bears a set of qualitative spatial relations to all other objects which it shares with no other object, the individuation of the reference frame cannot itself rely on spatial relations. Absolute theories of space avoid this possible circularity but at the expense of requiring an account of what it is that confers individuality on the points of space.

PII seeks to reduce the particular to the universal, the individual to a bundle of properties or attributes. But many philosophers have argued that such a reduction is not possible, that individuals involve something over and above their attributes, that confers individuation in an essential and unanalyzable way. This is to go the way of the Lockean substratum, the unknowable 'something' that attributes 'attach' to.

If an individual acquires its individuality by something that transcends its attributes we shall say that it exhibits <u>transcendental individuality</u>, TI for short[3]. Modern analytical philosophy has not taken kindly to TI. Physical individuals are usually regarded as being individuated by their location in space despite the problems just referred to (which may threaten to reintroduce TI for points of space!), and the problem of <u>reidentification</u>, of the grounds for claiming that an individual <u>b</u> at time t is the same individual as the individual <u>a</u> at an earlier time t', involves as necessary condition the spatio-temporal continuity of the trajectory joining the location of <u>a</u> at time t' and the location of <u>b</u> at time t. The proponents of TI might agree that spatio-temporal continuity (STC) is what allows us to <u>infer</u> a reidentification across an interval of time, but they would claim that it is the persistent TI, that, ontologically speaking, confers the reidentifiability.

2. <u>The Problem of Quantum Statistics</u>

In classical nineteenth century atomic physics the strongest version of PII, with attributes restricted to monadic intrinsic

properties, i.e. 'internal' properties independent of spatial location, is clearly false for the atoms. The weak version is however true, if we allow an impenetrability assumption (IA) to the effect that two distinct atoms can never occupy the same location in space. We want to begin our discussion of quantum physics by considering an argument to the effect that quantum particles cannot be regarded as individuals at all. If this were the case, the problem of how they are individuated simply would not arise. PII would not be either true or false but simply inapplicable. The argument runs like this. Consider the problem of distributing two quantum particles having the same intrinsic properties such as mass, spin and charge[4], but initially supposed to be individuals and labelled 1 and 2, among two possible pure quantum states $|a^r\rangle$ and $|a^s\rangle$, which we may suppose to be eigenstates of some maximal observable A for either particle with eigenvalues a^r and a^s as indicated by the notation for the states. (In what follows we assume for simplicity that all observables under discussion have a discrete spectrum). By analogy with the situation in classical

physics we might suppose that there are four possibilities:

(1) Both particles are in the state $|a^r\rangle$

(2) Both particles are in the state $|a^s\rangle$

(3) Particle 1 is in state $|a^r\rangle$ and particle 2 in state $|a^s\rangle$

(4) Particle 1 is in state $|a^s\rangle$ and particle 2 in state $|a^r\rangle$

Now in classical statistical mechanics arrangements 3 and 4 would be counted as distinct and given equal weight in assigning probabilities. But in quantum statistics, whether bosonic or fermionic, the arrangements 3 and 4 are counted as one and the same arrangement for the purpose of assigning weights. This is taken to show that the two arrangements are not only indistinguishable but are actually identical. But ontologically speaking these two arrangements are not identical if the two quantum particles are individuals. Hence the quantum particles cannot be individuals.

In passing we may note that this argument, while purporting to show that quantum particles fall outside the scope of PII, since they are not individuals, is also sometimes invoked to show that PII does apply to the states of affairs represented by the two arrangements 3 and 4[5].
What about arrangements 1 and 2? This is where bosons differ from fermions. For bosonic particles 1 and 2 are allowed arrangements to be counted with equal weight as compared with the single identified 3-cum-4 arrangement. For fermionic particles however arrangements 1 and 2 are not permitted at all. This is the famous Pauli Exclusion Principle.

We return in the next section to discuss the significance of this difference from the point of view of PII. But first we want to explain what is wrong with the argument concerning the identity of the arrangements 3 and 4. We begin by writing down the state vectors for the combined two-particle system corresponding to the arrangements 1,2,3 and 4. They are:

$$|a^r\rangle \otimes |a^r\rangle \qquad (1)$$

$$|a^s\rangle \otimes |a^s\rangle \qquad (2)$$

$$|a^r\rangle \otimes |a^s\rangle \qquad (3)$$

and

$$|a^s\rangle \otimes |a^r\rangle \qquad (4)$$

where we use the convention that in a tensor product of two states the left-hand member refers to particle 1 and the right-hand member to particle 2.
Now it is quite true that if the quantum particles are individuals then the states (3) and (4) are not identical. But the important point to notice is that these states are <u>not</u> the ones used in discussing quantum statistical mechanics. The relevant states for that purpose are as follows:

$$|a^r\rangle \otimes |a^r\rangle \tag{5}$$

$$|a^s\rangle \otimes |a^s\rangle \tag{6}$$

$$\frac{1}{\sqrt{2}}\left(|a^r\rangle \otimes |a^s\rangle + |a^s\rangle \otimes |a^r\rangle\right) \tag{7}$$

and $$\frac{1}{\sqrt{2}}\left(|a^r\rangle \otimes |a^s\rangle - |a^s\rangle \otimes |a^r\rangle\right) \tag{8}$$

The four states (5), (6), (7) and (8) are mutually orthogonal and span the same subspace as the states (1), (2), (3) and (4), but they are chosen so that (5), (6) and (7) are symmetric under exchange of particle labels (i.e. under exchange of left-hand and right-hand members of tensor products), while (8) is antisymmetric (changes sign) under the same operation.
Note that (5) and (6) are the same states as (1) and (2). The crucial difference is between the pairs (3) and (4) and (7) and (8).
Now (7) is no more identical with (8) than is (3) with (4).
But for bosons the states are restricted to the three <u>symmetric</u> possibilities. That is why (8) gets eliminated from the counting procedure, <u>not</u> because it gets identified with (7).
Similarly for fermions the states are restricted to the antisymmetric possibilities. But in this simple example this eliminates (5), (6) and (7), so (8) alone gets counted, but again not because it gets identified with (7).
To put the matter another way, states with the wrong symmetry get eliminated because they are not <u>accessible</u> to the joint quantum system, not because there are no such states!
It should be remembered that for time-evolution under a symmetric Hamiltonian[6] the symmetry character of a state cannot change with time, so no transitions can occur between symmetric bosonic states and antisymmetric fermionic states.

The upshot of our argument is to show, not that quantum particles must be individuals but rather that it is possible for them to be individuals, despite the peculiarities of quantum statistics. It is quite true that in quantum field theory (QFT) particles are not regarded as individuals. They are simply (quantized) excitations of a field. Particle labels do not enter into the discussion at all.

If our simple problem of counting the number of states for a two-particle system distributed over two one-particle states were transposed to quantum field theory, then for a bosonic (commuting) field there would be just three states corresponding to a double excitation of either state (mode) plus a single excitation of both states (modes). Similarly for a fermionic (anticommuting) field, there is only one state since double excitations are not allowed. So the quantum statistics comes out the way we want it to.

It is also true that there are strong arguments for regarding the 'quantized excitation' view of quantum particles as the correct one[7]. However, for the purposes of this paper, we continue to envisage the possibility of treating quantum particles as individuals and proceed to discuss whether they would or would not obey PII.

3. The Indistinguishability Postulate

What do we mean by saying that two quantum particles of the same species (characterized by their intrinsic properties) are indistinghuishable?

In quantum mechanics (QM) this is expressed by the Indistinguishability Postulate (IP)

ϕ = Greek phi

$$\langle P\phi | Q | P\phi \rangle = \langle \phi | Q | \phi \rangle, \forall Q, \forall \phi \qquad (9)$$

where $|\phi\rangle$ is an arbitrary N-particle state and Q a possible observable on the N-fold tensor product space of states. $|P\phi\rangle$ is an abreviation for $P|\phi\rangle$ where P is the unitary operator which is associated with an arbitrary permutation of the particle labels.

(9) says that it is not possible to tell by measuring the expectation value of any observable, whether the state of the system is $|\phi\rangle$ or $|P\phi\rangle$.

We notice that a sufficient condition for (9) to hold is that $|P\phi\rangle = \pm|\phi\rangle$ with Q any self-adjoint operator on the N-particle state-space. This interprets (9) as a restriction on the possible states for the N-particle system, allowing just the boson or fermion possibility. (Note that the choice of signs needs only to be made for transpositions, since any permutation can be represented as a product of transpositions, so even permutations are always associated with the plus sign, the distinction between bosons and fermions only arising for odd permutations).
But Greenberg and Messiah ([1964]) pointed out that (9) should, on a more profound analysis, be interpreted not as a restriction on states, but as a restriction on the possible observables for the N-particle system. On this view (9) can easily be shown to imply $P^{-1} Q P = Q$ or $QP = PQ$, so any permitted Q must commute with any permutation P. This in turn implies that Q must be a symmetric function of the particle labels. The label permutations provide effectively a set of non-Abelian superselecting operators, which can be used to resolve the state space into non-combining sectors associated with irreducible representations of the symmetric group S_N.
For two particles there are only two irreducible representations of S_2, provided by states which are symmetric or antisymmetric in the particle labels. So we are back to the boson and fermion possibilities but, with more than two particles, higher-dimensional representations of the symmetric group exist, allowing for the possibility of so-called parastatistics intermediate in character between bosonic and fermionic behaviour.

But even in the two-particle case it should be noted that the Messiah and Greenberg approach does not restrict the available states, only their accessibility in the way we described in the previous section.
Let Q now denote a possible observable (self-adjoint operator) on a single particle. Considered as possible physical magnitudes for the joint system we have two possibilities: $Q \otimes I$ for particle 1 having the property Q and $I \otimes Q$ for particle 2 having the property Q. Denote by $Q \otimes I$ by Q_1 and $I \otimes Q$ by Q_2. Then IP says, on the Messiah-Greenberg interpretation, that although Q_1 and Q_2 are self-adjoint operators on the Hilbert

space for the joint system, they cannot actually be observed. The intuition here is that observing Q_1 or Q_2 would involve knowing empirically which particle was which, and this is impossible if the particles are indistinguishable.
But from the point of view of discussing PII, it seems clear that we should not restrict the discussion to attributes which can actually be observed. This would restrict the discussion to symmetric combinations such as $Q_1 + Q_2$. The *ontological* significance of PII can only be brought out by discussing whether particles 1 and 2 have the same physical attributes expressed by Q_1 and Q_2 and their associated 'actualization' probabilities, while recognizing that these attributes can never be observed.[8] This is the task we shall attempt in the next section.

4. Quantum Individuals and the Identity of Indiscernibles

We begin by discussing the case of fermions. It has been claimed in the literature that the Pauli Exclusion Principle, prohibiting two fermionic particles from being in the same quantum state, is a clear vindication of PII[9]. What is being prohibited, apparently, is that the two fermions shall have both the same intrinsic state-independent properties of mass, spin, electric charge, etc and the same state-dependent properties expressed by expectation values of all quantum-mechanical physical magnitudes. But look at the allowed state (8). It is not true in such a state that each particle is present in a different state. *Each* particle clearly 'partakes' of *both* the states $|a^r\rangle$ and $|a^s\rangle$ in the superposition of product states expressed in (8). So might it not appear that in the *allowed* state both particles also have the *same* state-dependent properties, which would contradict PII? Let us formulate the state-dependent properties in terms of physical magnitudes such as Q_1 and Q_2 pertaining to each particle separately as discussed in the preceding section. In orthodox interpretations of quantum mechanics the properties Q_1 and Q_2 must be interpreted not as possessed values, but as propensities to yield specified 'actualization' results in accordance with the familiar statistical algorithm for computing the associated probabilities.

Ψ = Capital Greek psi
α = greek alpha
β = greek beta

Denoting the fermion state (8) by $|\Psi\rangle$ we shall be interested in comparing both monadic properties of the form $\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha)$ and $\text{Prob}^{|\Psi\rangle}(Q_2 = q^\alpha)$, where the notation indicates the probability in the state $|\Psi\rangle$ that the physical magnitude pertaining to either particle actualizes with the indicated value, and also relational properties of the form $\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha / Q_2 = q^\beta)$ and $\text{Prob}^{|\Psi\rangle}(Q_2 = q^\alpha / Q_1 = q^\beta)$ which refer to the conditional probabilities of actualizing one magnitude given the actualization result for the other.

These quantities are easily computed from the joint distribution

$$
\begin{aligned}
&\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha \,\&\, Q_2 = q^\beta) \\
&= |\langle q^\alpha | \langle q^\beta | (|\Psi\rangle)|^2 \\
&= \tfrac{1}{2} |\langle q^\alpha | a^r\rangle|^2 \cdot |\langle q^\beta | a^s\rangle|^2 \\
&+ \tfrac{1}{2} |\langle q^\alpha | a^s\rangle|^2 \cdot |\langle q^\beta | a^r\rangle|^2 \\
&- \text{Re}\, \langle a^r | q^\alpha\rangle\langle q^\alpha | a^s\rangle\langle a^s | q^\beta\rangle\langle q^\beta | a^r\rangle
\end{aligned} \tag{10}
$$

Summing this result over α and β to obtain the marginal probabilities and remembering $\sum_n |q^n\rangle\langle q^n| = I$, $\langle a^r | a^s\rangle = 0$ and $\langle a^r | a^r\rangle = \langle a^s | a^s\rangle = 1$ yields immediately

$$
\begin{aligned}
&\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha) = \text{Prob}^{|\Psi\rangle}(Q_2 = q^\alpha) \\
&= \tfrac{1}{2} |\langle q^\alpha | a^r\rangle|^2 + \tfrac{1}{2} |\langle q^\alpha | a^s\rangle|^2
\end{aligned} \tag{11}
$$

Similarly we find

$$
\begin{aligned}
&\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha / Q_2 = q^\beta) \\
&= \text{Prob}^{|\Psi\rangle}(Q_2 = q^\alpha / Q_1 = q^\beta) \\
&= [\,|\langle q^\alpha | a^r\rangle|^2 \cdot |\langle q^\beta | a^s\rangle|^2 + |\langle q^\alpha | a^s\rangle|^2 \cdot |\langle q^\beta | a^r\rangle|^2 \\
&- 2\text{Re}\langle a^r | q^\alpha\rangle\langle q^\alpha | a^s\rangle\langle a^s | q^\beta\rangle\langle q^\beta | a^r\rangle] \\
&/ [\,|\langle q^\beta | a^r\rangle|^2 + |\langle q^\beta | a^s\rangle|^2\,]
\end{aligned} \tag{12}
$$

The significance of (11) and (12) is that the two fermions in the state (8) do in fact have the same monadic properties and the same relational properties one to another, so the weakest form of PII which we can formulate which involves both monadic properties <u>and</u> relational properties, is violated.[10]

There are a number of comments we want to make concerning this conclusion and the way it was derived.

(1) In classical physics the state-dependent properties of a particle are completely specified by the maximally specific state description (location in phase space). Hence we can replace the question, "Do classical particles have the same state-dependent properties?" with the question "Do the two particles have the same maximally specific state description?"
If we try the same move in quantum mechanics we run into the problem that for a so-called 'entangled' state such as (8) there are no pure states which can be ascribed to the separate particles. (If there were such states the state of the combined system would be the tensor product of the states in question, but (8) is not of the form of a tensor product - it is a superposition of tensor products).
Now pure states in QM play the role of maximally specific states, so if we identified the relevant properties of the two particles with the pure states they are in, we would have to conclude that there is no answer to the question "Do they have the same properties?"

A corrolary of this result is that insofar as we can speak of states for the separate particles at all we must speak of mixed states[11]. Indeed the relevant mixed states are the same for the two particles[12] - equiprobable mixtures of the states $|a^r\rangle$ and $|a^s\rangle$. This is, of course, the essential content of the result for the marginal probability distributions for Q_1 and Q_2. But our analysis has gone beyond that involving the (improper) mixed states of the separate particles, by considering also the relational conditional probabilities[13] given in (12).

(2) There is another sort of relational property we might consider, expressed by comparing

$$\text{Prob}\,(Q_2 = q^\alpha / Q_2 = q^\beta) = \delta_{\alpha\beta} \tag{13}$$

with Prob $(Q_1 = q^\alpha / Q_2 = q^\beta)$ given by (12).
These relational properties of particle 2 to itself as compared with relations of particle 1 to particle 2 we reject as vindication of PII, for the same argument as we discussed in section 1 for ruling out illegitimate trivializations of PII. The purported vindication of PII again depends on regarding (13) as a monadic property of particle 2 whereas it is in fact a relational property of particle 2 to itself, which is also true as a relation of particle 1 to itself.

(3) If we write $\alpha = \beta$ in equation (12) then we indeed find for fermions the result

$$\mathrm{Prob}^{|\Psi\rangle}(Q_1 = q^{\alpha} / Q_2 = q^{\alpha}) = 0$$

This show that if actualization of Q_2 gives a certain value then there is zero probability that a concurrent actualization of Q_1 will yield the same value. This is the real significance of the Exclusion Principle, but has no bearing on PII, if we adhere to the orthodox view that actualizations do not correspond to antecedently existing possessed values.

(4) This brings us to our final comment. In hidden-variable reconstructions of QM, the circumstance demonstrated in point 3 above, would lead us to regard PII as vindicated for fermions (assuming that actualization results merely reveal preexisting values).

We now turn to the case of bosons. It is often assumed that purported violation of PII depends on consideration of states such as (5) or (6) where both particles can indeed be attributed the same pure state[14].

Φ = capital greek phi

Denoting the state (5) by $|\Phi\rangle$, for example, we can easily obtain the following results corresponding to (11) and (12):

$$\mathrm{Prob}^{|\Phi\rangle}(Q_1 = q^{\alpha}) = \mathrm{Prob}^{|\Phi\rangle}(Q_2 = q^{\alpha}) = |\langle q^{\alpha}|a^r\rangle|^2 \qquad (11')$$

and $$\mathrm{Prob}^{|\Phi\rangle}(Q_1 = q^{\alpha}/Q_2 = q^{\beta}) = \mathrm{Prob}^{|\Phi\rangle}(Q_2 = q^{\alpha}/Q_1 = q^{\beta}) = |\langle q^{\alpha}|a^r\rangle|^2 \qquad (12')$$

So, as we might expect, both monadic and relational properties are the same for the two particles.

But it should be noted that this conclusion is also true for the state (7), where two different states are involved. In this case the results (11) and (12) apply with the minus sign in front of the 'interference' term in (12) replaced by a plus sign.

Finally, we make a brief comment on the case of paraparticles. Here there do exist states for which the monadic properties of all the separate particles are not the same, but equally there are possible paraparticle states for which PII is violated in the same way as for bosons and fermions.

As an example consider the following normalized state for three parabosons of order 2^{15}:

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}(|a^r\rangle|a^r\rangle|a^s\rangle - |a^s\rangle|a^r\rangle|a^r\rangle) \qquad (14)$$

where $|a^r\rangle$ and $|a^s\rangle$ are two distinct one-particle states and triple tensor products are written in the sequence of particle labels 1, 2 and 3.

Denoting $Q \otimes I \otimes I$ by Q_1, $I \otimes Q \otimes I$ by Q_2 and $I \otimes I \otimes Q$ by Q_3, we obtain for the triple joint distribution

γ = greek gamma.

$$\begin{aligned}
&\mathrm{Prob}^{|\Psi'\rangle}(Q_1 = q^\alpha \,\&\, Q_2 = q^\beta \,\&\, Q_3 = q^\gamma) \\
&= |\langle q^\alpha| \otimes \langle q^\beta| \otimes \langle q^\gamma|(|\Psi\rangle)|^2 \\
&= \tfrac{1}{2}\Big[|\langle q^\alpha|a^r\rangle|^2 \cdot |\langle q^\beta|a^r\rangle|^2 \cdot |\langle q^\gamma|a^s\rangle|^2 \\
&+ |\langle q^\alpha|a^s\rangle|^2 \cdot |\langle q^\beta|a^r\rangle|^2 \cdot |\langle q^\gamma|a^r\rangle|^2 \\
&- 2\mathrm{Re}\langle a^r|q^\alpha\rangle\langle q^\alpha|a^s\rangle \langle a^r|q^\beta\rangle\langle q^\beta|a^r\rangle \langle a^s|q^\gamma\rangle\langle q^\gamma|a^r\rangle\Big]
\end{aligned} \qquad (15)$$

From (15) we find immediately the marginal distributions

$$\begin{aligned}
&\mathrm{Prob}^{|\Psi'\rangle}(Q_1 = q^\alpha) = \mathrm{Prob}^{|\Psi'\rangle}(Q_3 = q^\alpha) \\
&= \tfrac{1}{2}(|\langle q^\alpha|a^r\rangle|^2 + |\langle q^\alpha|a^s\rangle|^2)
\end{aligned} \qquad (16)$$

while

$$\mathrm{Prob}^{|\Psi'\rangle}(Q_2 = q^\alpha) = |\langle q^\alpha|a^r\rangle|^2 \qquad (17)$$

Thus particles 1 and 3 have the same monadic properties expressed by the marginal distributions but these differ from the monadic properties of particle 2.

Let us now show that particles 1 and 3 also have the same relational properties with respect to both the remaining particles.

We easily find that

$$\begin{aligned}
&\mathrm{Prob}^{|\Psi'\rangle}(Q_1 = q^\alpha / Q_3 = q^\gamma) = \mathrm{Prob}^{|\Psi'\rangle}(Q_3 = q^\alpha / Q_1 = q^\gamma) \\
&= \Big[|\langle q^\alpha|a^r\rangle|^2 \cdot |\langle q^\gamma|a^s\rangle|^2 + |\langle q^\alpha|a^s\rangle|^2 \cdot |\langle q^\gamma|a^r\rangle|^2 \\
&- 2\mathrm{Re}\langle a^r|q^\alpha\rangle\langle q^\alpha|a^s\rangle\langle a^s|q^\gamma\rangle\langle q^\gamma|a^r\rangle\Big] \\
&/\Big[|\langle q^\alpha|a^r|^2 + |\langle q^\alpha|a^s\rangle|^2\Big]
\end{aligned} \qquad (18)$$

Furthermore

$$\mathrm{Prob}_{|\Psi'\rangle}(Q_1 = q^{\alpha}/Q_2 = q^{\beta})$$
$$= \mathrm{Prob}_{|\Psi'\rangle}(Q_3 = q^{\alpha}/Q_2 = q^{\beta})$$
$$= \tfrac{1}{2}[\,|\langle q^{\alpha}|a^r\rangle|^2 + |\langle q^{\alpha}|a^s\rangle|^2] \qquad (19)$$

and finally

$$\mathrm{Prob}_{|\Psi'\rangle}(Q_1 = q^{\alpha}/Q_2 = q^{\beta} \,\&\, Q_3 = q^{\gamma})$$
$$= \mathrm{Prob}_{|\Psi'\rangle}(Q_3 = q^{\alpha}/Q_2 = q^{\beta} \,\&\, Q_1 = q^{\gamma})$$
$$= [\,|\langle q^{\alpha}|a^r\rangle|^2 \cdot |\langle q^{\beta}|a^s\rangle|^2 \cdot |\langle q^{\gamma}|a^r\rangle|^2$$
$$+ |\langle q^{\alpha}|a^s\rangle|^2 \cdot |\langle q^{\beta}|a^r\rangle|^2 \cdot |q^{\gamma}|a^r\rangle|^2$$
$$- 2\mathrm{Re}\langle a^r|q^{\alpha}\rangle\langle q^{\alpha}|a^s\rangle\langle a^s|q^{\beta}\rangle\langle q^{\beta}|a^r\rangle\langle a^r|q^{\gamma}\rangle\langle q^{\gamma}|a^r\rangle] \Big/$$
$$|\langle q^{\beta}|a^r\rangle|^2[\,|\langle q^{\gamma}|a^r\rangle|^2 + |\langle q^{\gamma}|a^s\rangle|^2] \qquad (20)$$

These results show that PII is violated for particles 1 and 3 in the state $|\Psi'\rangle$, even in its weakest form, that includes all the relevant relational properties.

5. Conclusion

There are two main conclusions of this paper. Firstly that indistinguishable particles in QM can be treated as individuals, but secondly, if they are so treated, then, on the most plausible reading of what constitutes a property of a quantal particle, even the weakest form of PII, including both monadic and relational properties, is violated both for bosons and fermions, and indeed for higher-order paraparticles.

It should be noted that if quantal particles are individuals, then their individuality must be conferred by TI. STC is not in general available in QM, since particles do not move in well-defined trajectories, so the question of spatio-temporal continuity of trajectory does not arise. The only exception to this is where the one-particle states involve well-defined wave packets, which diffuse sufficiently slowly

over time, as would be possible for the classical limit of sufficiently massive particles.
But it is clear that in the case of macroscopic bodies, where STC can be used to label the bodies, the STC criterion actually conflicts with the TI individuation of the elementary particles composing the body. To be strict every electron, for example, partakes of the state of every other electron in the universe, according to the antisymmetrization requirement!
But notice, that under conditions where the 'interference' term in (10) can be neglected, then the state $|\Psi>$ behaves like a proper mixture of states in which particle 1 is in state $|a^r>$ and particle 2 in state $|a^s>$ and the permuted state in which particle 1 is in state $|a^s>$ and particle 2 in state $|a^r>$, with equiprobable weights for the two component states in the mixture. So under these conditions the state (8) behaves like an equiprobable mixture of the states (3) and (4). In other words, when 'interference' can be neglected, we recover the same possibilities for states as in classical physics, where states (1) and (2) would anyway be eliminated by IA assuming them to be maximally specific.
But, of course, ontologically speaking, 'interference' is never strictly absent. That, after all, is what constitutes the 'problem of measurement' in QM, so the involvement of every electron with the state of every other electron in the universe, although negligible for practical purposes, remains an ontological commitment of QM, under the interpretation where the particles are treated as individuals.
If this sounds too bizarre to be acceptable, it provides another argument for preferring the treatment of indistinguishable particles along the lines provided by quantum field theory.

In this paper we have been concerned with conceptual possibilities, rather than what is most reasonable to believe about the ontological status of elementary particles.

Notes

* Some of the arguments in this paper appeared in a thesis submitted by one of us (S.F.) in partial fulfillment of the requirements for the PhD degree of the University of London, in 1984, entitled "Identity and Individuality in Classical and Quantum Physics".

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1. There is much discussion in the literature as to whether a clear-cut distinction can be made between relational and monadic properties. For useful comments see Hoy [1984].

2. There is no real concensus about Leibniz's own views on the status of PII. The locus classicus is generally held to be Leibniz (1961) Book II Chapter XXVII. For an influential critical discussion see in particular Ishiguro [1976].

3. This terminology is due to Post [1963].

4. In the physics literature such particles are often referred as 'identical'. In our terminology this would mean they were one and the same particle! We shall use the term 'indistinguishable', in place of the physicist's 'identical'.

5. This interpretation of PII is mooted in Lucas [1984] p.131.

6. The justification for assuming that the Hamiltonian observable must be a symmetric function of the particle labels will become apparent in Section 3.

7. See Redhead [1983] and [1986] for further discussion of the QFT approach.

8. We refer to 'actualization' rather than 'measurement result', to emphasize the point that they are not observable. They will, however, be produced by measurement interactions, but not in a way which makes them identifiable as contrasted with their particle label permuted variants. But note that the probability of observing some eigenvalue q^{α} for Q on one or other particle is calculated in QM as the sum Prob $(Q_1 = q^{\alpha})$ + Prob $(Q_2 = q^{\alpha})$.
α = greek alpha
   We are claiming here ontological significance for each individual summand in this formula for an observable probability.

9. See Shadmi [1978].

10. This result remains true if we consider also relational properties of the form $\mathrm{Prob}^{\Psi}(Q_1' = q^\alpha / Q_2' = q^\beta)$ which is easily seen to be equal to $\mathrm{Prob}^{\Psi}(Q_2' = q^\alpha / Q_1' = q^\beta)$ where $Q_1' = Q' \otimes I$, $Q_2' = I \otimes Q'$ and Q' is a self-adjoint operator <u>distinct</u> from Q. This generalization also applies to the other violations of PII discussed in this section.

11. See D'Espagnat [1976] pp.58-61 for a discussion of mixed states arising in this way. He calls them 'improper' mixtures.

12. A similar point is made in van Fraassen [1984]. See also Margenau [1944] and [1950].

13. It should be stressed that these relational properties expressed by the conditional probabilities in no way supervene on the monadic properties expressed by the marginal distributions. In the terminology of Teller [1986] they are <u>inherent</u> relations.

14. See, for example, the discussion of PII for bosons given by Cortes [1976], Barnette [1978], Ginsberg [1981] and Teller [1983].

15. Compare Hartle and Taylor [1969] for details of how to construct paraparticle states. The state (14) is obtained from their (2.5) by identifying two of the one-particle states.

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